Chapter 1

Econometrics

There are no exercises or applications in Chapter 1.

**Example 1.2**

**import$**

**Year, X,C**

**2000, 8559.4, 6830.4**

**2001, 8883.3, 7148.8**

**2002, 9060.1, 7439.2**

**2003, 9378.1, 7804.0**

**2004, 9937.2, 8285.1**

**2005, 10485.9, 8819.0**

**2006, 11268.1, 9322.7**

**2007, 11894.1, 9826.4**

**2008, 12238.8, 10129.9**

**2009, 12030.3, 10088.5**

**plot**

**;lhs=x**

**;rhs=c**

**;limits=6500,10500**

**;endpoints=8500,12500**

**;grid**

**;regression**

**;vaxis=Personal Consumption;Footer=Personal Income**

**;Title=Figure 1.1 Aggregate U.S. Consumption and Income Data, 2000-2009$**



(Dates were added to the figure by editing.)

Chapter 2

The Linear Regression Model

There are no exercises or applications in Chapter 2.

**Example 2.1. Keynes’s Consumption**

**import$**

**Year X C W**

**1940 241 226 0**

**1941 280 240 0**

**1942 319 235 1**

**1943 331 245 1**

**1944 345 255 1**

**1945 340 265 1**

**1946 332 295 0**

**1947 320 300 0**

**1948 339 305 0**

**1949 338 315 0**

**1950 371 325 0**

**plot;lhs=x;rhs=c;limits=200,350; endpoints=225,375;regression**

**;title=Figure 2.1 Consumption Data, 1940-1950 $**



(Dates and dashed lines were added by editing.)

**Example 2.7. Nonzero Conditional Mean of the Disturbances**



Chapter 3

Least Squares Regression

**EXAMPLES – Section 3.2.2 and Table 3.2**

**Import$**

**YEAR RealGNP Invest GNPDefl Interest Infl Trend RealInv**

**2000 87.1 2.034 81.9 9.23 3.4 1 2.484**

**2001 88.0 1.929 83.8 6.91 1.6 2 2.311**

**2002 89.5 1.925 85.0 4.67 2.4 3 2.265**

**2003 92.0 2.028 86.7 4.12 1.9 4 2.339**

**2004 95.5 2.277 89.1 4.34 3.3 5 2.556**

**2005 98.7 2.527 91.9 6.19 3.4 6 2.750**

**2006 101.4 2.681 94.8 7.96 2.5 7 2.828**

**2007 103.2 2.644 97.3 8.05 4.1 8 2.717**

**2008 102.9 2.425 99.2 5.09 0.1 9 2.445**

**2009 100.0 1.878 100.0 3.25 2.7 10 1.878**

**2010 102.5 2.101 101.2 3.25 1.5 11 2.076**

**2011 104.2 2.240 103.3 3.25 3.0 12 2.168**

**2012 105.6 2.479 105.2 3.25 1.7 13 2.356**

**2013 109.0 2.648 106.7 3.25 1.5 14 2.482**

**2014 111.6 2.856 108.3 3.25 0.8 15 2.637**

**EndData**

**Create ; Y = RealInv $**

**Create ; T = trend $**

**Create ; G = realgnp $**

**Create ; R = interest $**

**Create ; P = infl $**

**Namelist;z=y,t,g,r,p$**

**Dstat ; rhs=z$**

**--------+---------------------------------------------------------------------**

 **| Standard Missing**

**Variable| Mean Deviation Minimum Maximum Cases Values**

**--------+---------------------------------------------------------------------**

 **Y| 2.420067 .262666 1.878 2.828 15 0**

 **T| 8.0 4.472136 1.0 15.0 15 0**

 **G| 99.41333 7.525468 87.1 111.6 15 0**

 **R| 5.070667 2.081351 3.25 9.23 15 0**

 **P| 2.26 1.092703 .1 4.1 15 0**

**--------+---------------------------------------------------------------------**

**Descriptive Statistics for 5 variables**

**Dstat results are matrix LASTDSTA in current project.**

**Regress;Lhs=y;rhs=one,t,g,r,p$**

**-----------------------------------------------------------------------------**

**Ordinary least squares regression ............**

**LHS=Y Mean = 2.42007**

 **Standard deviation = .26267**

**---------- No. of observations = 15 DegFreedom Mean square**

**Regression Sum of Squares = .760908 4 .19023**

**Residual Sum of Squares = .205002 10 .02050**

**Total Sum of Squares = .965911 14 .06899**

**---------- Standard error of e = .14318 Root MSE .11691**

**Fit R-squared = .78776 R-bar squared .70287**

**Model test F[ 4, 10] = 9.27926 Prob F > F\* .00213**

**--------+--------------------------------------------------------------------**

 **| Standard Prob. 95% Confidence**

 **Y| Coefficient Error t |t|>T\* Interval**

**--------+--------------------------------------------------------------------**

**Constant| -6.26176\*\*\* 1.93671 -3.23 .0090 -10.57700 -1.94651**

 **T| -.16187\*\*\* .04739 -3.42 .0066 -.26746 -.05628**

 **G| .09960\*\*\* .02421 4.11 .0021 .04566 .15355**

 **R| .01972 .03380 .58 .5725 -.05559 .09503**

 **P| -.01109 .03990 -.28 .7867 -.09998 .07781**

**--------+--------------------------------------------------------------------**

**\*\*\*, \*\*, \* ==> Significance at 1%, 5%, 10% level.**

**Model was estimated on Aug 01, 2017 at 08:37:09 AM**

**-----------------------------------------------------------------------------**

**Namelist; x=one,t,g,r,p$**

**Matrix ; list;x'x$**

**--------+----------------------------------------------------------------------**

 **RESULT| 1 2 3 4 5**

**--------+----------------------------------------------------------------------**

 **1| 15.0000 120.000 1491.20 76.0600 33.9000**

 **2| 120.000 1240.00 12381.5 522.060 244.100**

 **3| 1491.20 12381.5 149038. 7453.03 3332.83**

 **4| 76.0600 522.060 7453.03 446.323 186.656**

 **5| 33.9000 244.100 3332.83 186.656 93.3300**

**Matrix ; list;x'y$**

**--------+--------------**

 **RESULT| 1**

**--------+--------------**

 **1| 36.3010**

 **2| 288.691**

 **3| 3612.90**

 **4| 188.300**

 **5| 82.8193**

**Matrix ; list;<x'x>\*x'y$**

**--------+--------------**

 **RESULT| 1**

**--------+--------------**

 **1| -6.26176**

 **2| -.161870**

 **3| .0996027**

 **4| .0197220**

 **5| -.0110883**

**Matrix ; list;xcor(z)$**

**--------+--------------------------------------------**

**Cor.Mat.| Y T G R P**

**--------+--------------------------------------------**

 **Y| 1.00000 -.10441 .14809 .55261 .19388**

 **T| -.10441 1.00000 .95910 -.66317 -.39612**

 **G| .14809 .95910 1.00000 -.49410 -.32384**

 **R| .55261 -.66317 -.49410 1.00000 .46358**

 **P| .19388 -.39612 -.32384 .46358 1.00000**

**Create ; dy = dev(y) $**

**Create ; dt = dev(t) $**

**Create ; dg = dev(g) $**

**Calc ; list ; xbr(y)**

 **; xbr(t)**

 **; xbr(g) $**

**[CALC] = 2.4200667**

**[CALC] = 8.0000000**

**[CALC] = 99.4133333**

**Calculator: Computed 3 scalar results**

**Calc ; list ; sty = dt'dy**

 **; sgg = dg'dg**

 **; sgy = dg'dy**

 **; stg = dt'dg**

 **; stt = dt'dt$**

**[CALC] STY = -1.7170000**

**[CALC] SGG = 792.8573333**

**[CALC] SGY = 4.0982867**

**[CALC] STG = 451.9000000**

**[CALC] STT = 280.0000000**

**Calculator: Computed 5 scalar results**

**Calc ; list ; b2 = (sty\*sgg - sgy\*stg)/(stt\*sgg-stg\*stg)$**

**[CALC] B2 = -.1806630**

**Calc ; list ; b3 = (sgy\*stt - sty\*stg)/(stt\*sgg-stg\*stg)$**

**[CALC] B3 = .1081404**

**Calc ; list ; b1 = xbr(y) - b2\*xbr(t)-b3\*xbr(g)$**

**[CALC] B1 = -6.8852242**

**Calc ; list ; byg = sgy / sgg $**

**[CALC] BYG = .0051690**

**Calc ; list ; byt = sty / stt $**

**[CALC] BYT = -.0061321**

**Calc ; list ; btg = stg / sgg$**

**[CALC] BTG = .5699638**

**Calc ; list ; r2gt=stg^2/(sgg\*stt)$**

**[CALC] R2GT = .9198809**

**Calc ; list ; byg\_t=byg-((byt\*btg-r2gt\*byg)/(1-r2gt))$**

**[CALC] BYG\_T = .1081404**

**Namelist ; yvar=y $**

**Matrix;list;xcor(x,yvar)$**

**--------+--------**

**Cor.Mat.| Y**

**--------+--------**

 **ONE| .00000**

 **T| -.10441**

 **G| .14809**

 **R| .55261**

 **P| .19388**

**Regress;quietly ; Lhs=y;rhs=one,t,g,r,p$**

**Matrix ; vars = diag(varb) ; sdevs=sqrt(vars)$**

**Matrix ; tstats = <sdevs>\*b$**

**Matrix ; pcor = dirp(tstats,tstats) + degfrdm$**

**Matrix ; pci = diri(pcor)$**

**Matrix ; pcor = dirp(tstats,tstats,pci)$**

**Matrix ; list ; pcor = esqr(pcor)$**

**--------+--------------**

 **PCOR| 1**

**--------+--------------**

 **1| .000000**

 **2| .733814**

 **3| .792847**

 **4| .181449**

 **5| .0875491**

**Exercises**

1. Let .

(a) The normal equations are given by (3-12), (we drop the minus sign), hence for each of the columns of **X**, **x***k*, we know that **x***k*′**e** = 0. This implies that and.

(b) Use  to conclude from the first normal equation that .

(c) We know that  and . It follows then that because

 . Substitute *ei* to obtain 

or 

Then, 

(d) The first derivative vector of **e′e** is -2**X′e**. (The normal equations.) The second derivative matrix is

∂2(**e′e**)/∂**b**∂**b′** = 2**X′X**. We need to show that this matrix is positive definite. The diagonal elements are 2*n* and which are clearly both positive. The determinant is

[(2*n*)( )] - ()2 = -4()2 = .

Note that a much simpler proof appears after (3-6).

2. Write **c** as **b** + (**c** ‑ **b**). Then, the sum of squared residuals based on **c** is

(**y** ‑ **Xc**)**′**(**y** ‑ **Xc**) = [**y** ‑ **X**(**b** + (**c** ‑ **b**))] **′**[**y** ‑ **X**(**b** + (**c** ‑ **b**))] = [(**y** ‑ **Xb**) + **X**(**c** ‑ **b**)] **′**[(**y** ‑ **Xb**) + **X**(**c** ‑ **b**)]

 = (**y** ‑ **Xb**) **′**(**y** ‑ **Xb**) + (**c** ‑ **b**) **′X′X**(**c** ‑ **b**) + 2(**c** ‑ **b**) **′X′**(**y** ‑ **Xb**).

But, the third term is zero, as 2(**c** ‑ **b**) **′X′**(**y** ‑ **Xb**) = 2(**c** ‑ **b**)**X′e** = **0**. Therefore,

 (**y** ‑ **Xc**) **′**(**y** ‑ **Xc**) = **e′e** + (**c** ‑ **b**) **′X′X**(**c** ‑ **b**)

or (**y** ‑ **Xc**) **′**(**y** ‑ **Xc**) ‑ **e′e** = (**c** ‑ **b**) **′X′X**(**c** ‑ **b**).

The right hand side can be written as **d′d** where **d** = **X**(**c** ‑ **b**), so it is necessarily positive. This confirms what we knew at the outset, least squares is least squares.

3. In the regression of **y** on **i** and **X**, the coefficients on **X** are **b** = (**X′M0X**)-1**X′M**0**y**. **M**0 = **I** ‑ **i**(**i′i**)-1**i′** is the matrix which transforms observations into deviations from their column means. Since **M**0 is idempotent and symmetric we may also write the preceding as [(**X′M**0**′**)(**M**0**X**)]-1(**X′M**0**′**)(**M**0**y**) which implies that the regression of **M**0**y** on **M**0**X** produces the least squares slopes. If only **X** is transformed to deviations, we would compute [(**X′M**0**′**)(**M**0**X**)]-1(**X′M**0**′**)**y** but, of course, this is identical. However, if only **y** is transformed, the result is (**X′X**)-1**X′M**0**y** which is likely to be quite different.

4. What is the result of the matrix product **M**1**M** where **M**1 is defined in (3‑19) and **M** is defined in (3‑14)?

 **M**1**M** = (**I** ‑ **X**1(**X**1**′X**1)-1**X**1**′**)(**I** ‑ **X**(**X′X**)-1**X′**) = **M** ‑ **X**1(**X**1**′X**1)-1**X**1**′M**

There is no need to multiply out the second term. Each column of **MX**1 is the vector of residuals in the regression of the corresponding column of **X**1 on all of the columns in **X**. Since that **x** is one of the columns in **X**, this regression provides a perfect fit, so the residuals are zero. Thus, **MX**1 is a matrix of zeroes which implies that **M**1**M** = **M**.

5. The original **X** matrix has *n* rows. We add an additional row, **x***s*′. The new **y** vector likewise has an additional element. Thus,  The new coefficient vector is

 **b***n,s* = (**X***n,s*′ **X***n,s*)-1(**X***n,s*′**y***n,s*). The matrix is **X***n,s*′**X***n,s* = **X***n***′X***n* + **x***s***x***s*′. To invert this, use (A -66);

 . The vector is

 (**X***n,s*′**y***n,s*) = (**X***n*′**y***n*) + **x***sys*. Multiply out the four terms to get

 (**X***n,s*′ **X***n,s*)-1(**X***n,s*′**y***n,s*) =

 **b***n* – +  **x***sys*  **x***sys*

=

 **b***n* +  **x***sys* – –

 **b***n* + –

 **b***n* + –

 **b***n* + 

6. Define the data matrix as follows:  (The subscripts on the parts of **y** refer to the “observed” and “missing” rows of **X**. We will use Frish-Waugh to obtain the first two columns of the least squares coefficient vector.  **b**1=(**X**1′**M**2**X**1)-1(**X**1′**M**2**y**). Multiplying it out, we find that **M**2 = an identity matrix save for the last diagonal element that is equal to 0.

**X**1′**M**2**X**1 = . This just drops the last observation. **X**1′**M**2**y** is computed likewise. Thus, the coeffients on the first two columns are the same as if *y*0 had been linearly regressed on **X**1. The denomonator of *R*2 is different for the two cases (drop the observation or keep it with zero fill and the dummy variable). For the first strategy, the mean of the *n*-1 observations should be different from the mean of the full *n* unless the last observation happens to equal the mean of the first *n*-1.

 For the second strategy, replacing the missing value with the mean of the other *n*-1 observations, we can deduce the new slope vector logically. Using Frisch-Waugh, we can replace the column of *x*’s with deviations from the means, which then turns the last observation to zero. Thus, once again, the coefficient on the *x* equals what it is using the earlier strategy. The constant term will be the same as well.

7. For convenience, reorder the variables so that **X** = [**i**, **P***d*, **P***n*, **P***s*, **Y**]. The three dependent variables are **E***d*, **E***n*, and **E***s*, and **Y** = **E***d* + **E***n* + **E***s*. The coefficient vectors are

 **b***d* = (**X′X**)-1**X′E***d*,

 **b***n* = (**X′X**)-1**X′E***n*, and

 **b***s* = (**X′X**)-1**X′E***s*.

The sum of the three vectors is

 **b** = (**X′X**)-1**X**′[**E***d* + **E***n* + **E***s*] = (**X′X**)-1**X**′**Y**.

Now, **Y** is the last column of **X**, so the preceding sum is the vector of least squares coefficients in the regression of the last column of **X** on all of the columns of **X**, including the last. Of course, we get a perfect fit. In addition, **X′**[**E***d* + **E***n* + **E***s*] is the last column of **X′X**, so the matrix product is equal to the last column of an identity matrix. Thus, the sum of the coefficients on all variables except income is 0, while that on income is 1.

8. Let  denote the adjusted *R*2 in the full regression on *K* variables including **x***k*, and letdenote the adjusted *R*2 in the short regression on *K*‑1 variables when **x***k* is omitted. Let and denote their unadjusted counterparts. Then,

= 1 ‑ **e′e**/**y′M**0**y**

= 1 ‑ **e**1**′e**1/**y′M**0**y**

where **e′e** is the sum of squared residuals in the full regression, **e**1**′e**1 is the (larger) sum of squared residuals in the regression which omits **x***k*, and **y′M**0**y** = Σ*i* (*yi* -)2.

Then,= 1 ‑ [(*n*‑1)/(*n*‑*K*)](1 ‑ )

and= 1 ‑ [(*n*‑1)/(*n*‑(*K*‑1))](1 ‑).

The difference is the change in the adjusted *R*2 when **x***k* is added to the regression,

- = [(*n*-1)/(*n*-*K*+1)][**e**1**′e**1/**y′M**0**y**] - [(*n*-1)/(*n*-*K*)][**e′e**/**y′M**0**y**].

The difference is positive if and only if the ratio is greater than 1. After cancelling terms, we require for the adjusted *R*2 to increase that **e**1**′e**1/(*n-K*+1)]/[(*n-K*)/**e′e**] > 1. From the previous problem, we have that **e**1**′e**1 = **e′e** + *bK2*(**x***k***′M**1**x***k*), where **M**1 is defined above and *bk* is the least squares coefficient in the full regression of **y** on **X**1 and **x***k*. Making the substitution, we require [(**e′e** + *bK2*(**x***k***′M**1**x***k*))(*n*-*K*)]/[(*n*-*K*)**e′e** + **e′e**] > 1. Since **e′e** = (*n*‑*K*)*s*2, this simplifies to [**e′e** + *bK2*(**x***k***′M**1**x***k*)]/[**e′e** + *s*2] > 1. Since all terms are positive, the fraction is greater than one if and only *bK2*(**x***k***′M**1**x***k*) > *s*2 or *bK2*(**x***k***′M**1**x***k*/*s*2) > 1. The denominator is the estimated variance of *bk*, so the result is proved.

9. This *R*2 must be lower. The sum of squares associated with the coefficient vector which omits the constant term must be higher than the one which includes it. We can write the coefficient vector in the regression without a constant as **c** = (0,**b**\*) where **b**\* = (**W′W**)-1**W′y**, with **W** being the other *K*‑1 columns of **X**. Then, the result of the previous exercise applies directly.

10. We use the notation ‘Var[.]’ and ‘Cov[.]’ to indicate the sample variances and covariances. Our information is Var[*N*] = 1, Var[*D*] = 1, Var[*Y*] = 1.

Since *C* = *N* + *D*, Var[*C*] = Var[*N*] + Var[*D*] + 2Cov[*N*,*D*] = 2(1 + Cov[*N*,*D*]).

From the regressions, we have

 Cov[*C*,*Y*]/Var[*Y*] = Cov[*C*,*Y*] = .8.

But, Cov[*C*,*Y*] = Cov[*N*,*Y*] + Cov[*D*,*Y*].

Also, Cov[*C*,*N*]/Var[*N*] = Cov[*C*,*N*] = .5,

but, Cov[*C*,*N*] = Var[*N*] + Cov[*N*,*D*] = 1 + Cov[*N*,*D*], so Cov[*N*,*D*] = ‑.5,

so that Var[*C*] = 2(1 + ‑.5) = 1.

And, Cov[*D*,*Y*]/Var[*Y*] = Cov[*D*,*Y*] = .4.

Since Cov[*C*,*Y*] = .8 = Cov[*N*,*Y*] + Cov[*D*,*Y*], Cov[*N*,*Y*] = .4.

Finally, Cov[*C*,*D*] = Cov[*N*,*D*] + Var[*D*] = ‑.5 + 1 = .5.

Now, in the regression of *C* on *D*, the sum of squared residuals is (*n*‑1){Var[*C*] ‑ (Cov[*C*,*D*]/Var[*D*])2Var[*D*]}

based on the general regression result Σ*e*2 = Σ(*yi* -**)2 ‑ *b*2Σ(*xi* -)2. All of the necessary figures were obtained above. Inserting these and *n*‑1 = 20 produces a sum of squared residuals of 15.

11. Computed results are

**Regress;lhs=realinv;rhs=one,realgnp,interest$**

**-----------------------------------------------------------------------------**

**Ordinary least squares regression ............**

**LHS=REALINV Mean = 2.42007**

 **Standard deviation = .26267**

**---------- No. of observations = 15 DegFreedom Mean square**

**Regression Sum of Squares = .521605 2 .26080**

**Residual Sum of Squares = .444305 12 .03703**

**Total Sum of Squares = .965911 14 .06899**

**---------- Standard error of e = .19242 Root MSE .17211**

**Fit R-squared = .54001 R-bar squared .46335**

**Model test F[ 2, 12] = 7.04388 Prob F > F\* .00947**

**--------+--------------------------------------------------------------------**

 **| Standard Prob. 95% Confidence**

 **REALINV| Coefficient Error t |t|>T\* Interval**

**--------+--------------------------------------------------------------------**

**Constant| -.04298 .86319 -.05 .9611 -1.92371 1.83775**

 **REALGNP| .01945\*\* .00786 2.47 .0293 .00232 .03657**

**INTEREST| .10448\*\*\* .02842 3.68 .0032 .04256 .16640**

**--------+--------------------------------------------------------------------**

**Namelist; X=one,realgnp,interest$**

**Matrix ; list ; x'x ; x'realinv$**

 **RESULT| 1 2 3**

**--------+------------------------------------------**

 **1| 15.0000 1491.20 76.0600**

 **2| 1491.20 149038. 7453.03**

 **3| 76.0600 7453.03 446.323**

 **RESULT| 1**

**--------+--------------**

 **1| 36.3010**

 **2| 3612.90**

 **3| 188.300**

**Matrix ; list ; <x'x>\*x'realinv$**

 **RESULT| 1**

**--------+--------------**

 **1| -.0429785**

 **2| .0194467**

 **3| .104480**

**Matrix ; list ; ba=<x'x>\*x'realinv$**

 **BA| 1**

**--------+--------------**

 **1| -.0429785**

 **2| .0194467**

 **3| .104480**

**Matrix ; e = realinv - x\*ba$**

**Calc ; list ; r2 = 1 - e'e / ((n-1)\*var(realinv)) $**

**[CALC] R2 = .5400140**

12.The results cannot be correct. Since log *S/N* = log *S/Y* + log *Y/N* by simple, exact algebra, the same result must apply to the least squares regression results. That means that the second equation estimated must equal the first one plus log *Y/N*. Looking at the equations, that means that all of the coefficients would have to be identical save for the second, which would have to equal its counterpart in the first equation, plus 1. Therefore, the results cannot be correct. In an exchange between Leff and Arthur Goldberger that appeared later in the same journal, Leff argued that the difference was simple rounding error. You can see that the results in the second equation resemble those in the first, but not enough so that the explanation is credible. Further discussion about the data themselves appeared in subsequent discussion. [See Goldberger (1973) and Leff (1973).]

13. a. Consider a regresion of y on x1, x2 and x3. The incremental contribution of x3 will be different depending on whether the order entered is (x1,x3,x2) or (x1,x2,x3), (x2,x1,x3), or (x2,x3,x1).

b. Use the equation above (3-31) and consider x2 after x1. If x1 and x2 are orthogonal, then **X**2’**M**1**X**2 = **X**2’**X**2 and the result reduces to R1.22 = R12 + R22. This is the if part. For only if, note that (3-31) implies that if the variables are not orthogonal, then, as observed earlier the previous result cannot hold.

c. Entering T first raises R2 from 0.00000 to 0.01090. Entering T last raises R2 from .54013 to .78776.

**-----------------------------------------------------**

**Ordinary least squares regression ............**

**T entered first R-squared = .01090**

**T not entered R-squared = .54013**

**T entered last R-squared = .78776**

**------------------------------------------------------**

**Application**

?=======================================================================

? Chapter 3 Application 1

?=======================================================================

Read $

(Data appear in the text.)

Namelist ; X1 = one,educ,exp,ability$

Namelist ; X2 = mothered,fathered,sibs$

?=======================================================================

? a.

?=======================================================================

Regress ; Lhs = wage ; Rhs = x1$

+----------------------------------------------------+

| Ordinary least squares regression |

| LHS=WAGE Mean = 2.059333 |

| Standard deviation = .2583869 |

| WTS=none Number of observs. = 15 |

| Model size Parameters = 4 |

| Degrees of freedom = 11 |

| Residuals Sum of squares = .7633163 |

| Standard error of e = .2634244 |

| Fit R-squared = .1833511 |

| Adjusted R-squared = -.3937136E-01 |

| Model test F[ 3, 11] (prob) = .82 (.5080) |

+----------------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |t-ratio |P[|T|>t]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

 Constant| 1.66364000 .61855318 2.690 .0210

 EDUC | .01453897 .04902149 .297 .7723 12.8666667

 EXP | .07103002 .04803415 1.479 .1673 2.80000000

 ABILITY | .02661537 .09911731 .269 .7933 .36600000

?=======================================================================

? b.

?=======================================================================

Regress ; Lhs = wage ; Rhs = x1,x2$

+----------------------------------------------------+

| Ordinary least squares regression |

| LHS=WAGE Mean = 2.059333 |

| Standard deviation = .2583869 |

| WTS=none Number of observs. = 15 |

| Model size Parameters = 7 |

| Degrees of freedom = 8 |

| Residuals Sum of squares = .4522662 |

| Standard error of e = .2377673 |

| Fit R-squared = .5161341 |

| Adjusted R-squared = .1532347 |

| Model test F[ 6, 8] (prob) = 1.42 (.3140) |

+----------------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |t-ratio |P[|T|>t]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

 Constant| .04899633 .94880761 .052 .9601

 EDUC | .02582213 .04468592 .578 .5793 12.8666667

 EXP | .10339125 .04734541 2.184 .0605 2.80000000

 ABILITY | .03074355 .12120133 .254 .8062 .36600000

 MOTHERED| .10163069 .07017502 1.448 .1856 12.0666667

 FATHERED| .00164437 .04464910 .037 .9715 12.6666667

 SIBS | .05916922 .06901801 .857 .4162 2.20000000

?=======================================================================

? c.

?=======================================================================

Regress ; Lhs = mothered ; Rhs = x1 ; Res = meds $

Regress ; Lhs = fathered ; Rhs = x1 ; Res = feds $

Regress ; Lhs = sibs ; Rhs = x1 ; Res = sibss $

Namelist ; X2S = meds,feds,sibss $

Matrix ; list ; Mean(X2S) $

Matrix Result has 3 rows and 1 columns.

 1

 +--------------

 1| -.1184238D-14

 2| .1657933D-14

 3| -.5921189D-16

The means are (essentially) zero. The sums must be zero, as these new variables are orthogonal to the columns of X1. The first column in X1 is a column of ones, so this means that these residuals must sum to zero.

?=======================================================================

? d.

?=======================================================================

Namelist ; X = X1,X2 $

Matrix ; i = init(n,1,1) $

Matrix ; M0 = iden(n) - 1/n\*i\*i' $

Matrix ; b12 = <X'X>\*X'wage$

Calc ; list ; ym0y =(N-1)\*var(wage) $

Matrix ; list ; cod = 1/ym0y \* b12'\*X'\*M0\*X\*b12 $

Matrix COD has 1 rows and 1 columns.

 1

 +--------------

 1| .51613

Matrix ; e = wage - X\*b12 $

Calc ; list ; cod = 1 - 1/ym0y \* e'e $

COD = .516134

The R squared is the same using either method of computation.

Calc ; list ; RsqAd = 1 - (n-1)/(n-col(x))\*(1-cod)$

RSQAD = .153235

? Now drop the constant

Namelist ; X0 = educ,exp,ability,X2 $

Matrix ; i = init(n,1,1) $

Matrix ; M0 = iden(n) - 1/n\*i\*i' $

Matrix ; b120 = <X0'X0>\*X0'wage$

Matrix ; list ; cod = 1/ym0y \* b120'\*X0'\*M0\*X0\*b120 $

Matrix COD has 1 rows and 1 columns.

 1

 +--------------

 1| .52953

Matrix ; e0 = wage - X0\*b120 $

Calc ; list ; cod = 1 - 1/ym0y \* e0'e0 $

COD = .515973

The R squared now changes depending on how it is computed. It also goes up, completely artificially.

?=======================================================================

? e.

?=======================================================================

The R squared for the full regression appears immediately below.

? f.

Regress ; Lhs = wage ; Rhs = X1,X2 $

+----------------------------------------------------+

| Ordinary least squares regression |

| WTS=none Number of observs. = 15 |

| Model size Parameters = 7 |

| Degrees of freedom = 8 |

| Fit R-squared = .5161341 |

+----------------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |t-ratio |P[|T|>t]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

 Constant| .04899633 .94880761 .052 .9601

 EDUC | .02582213 .04468592 .578 .5793 12.8666667

 EXP | .10339125 .04734541 2.184 .0605 2.80000000

 ABILITY | .03074355 .12120133 .254 .8062 .36600000

 MOTHERED| .10163069 .07017502 1.448 .1856 12.0666667

 FATHERED| .00164437 .04464910 .037 .9715 12.6666667

 SIBS | .05916922 .06901801 .857 .4162 2.20000000

Regress ; Lhs = wage ; Rhs = X1,X2S $

| Ordinary least squares regression |

| WTS=none Number of observs. = 15 |

| Model size Parameters = 7 |

| Degrees of freedom = 8 |

| Fit R-squared = .5161341 |

| Adjusted R-squared = .1532347 |

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |t-ratio |P[|T|>t]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

 Constant| 1.66364000 .55830716 2.980 .0176

 EDUC | .01453897 .04424689 .329 .7509 12.8666667

 EXP | .07103002 .04335571 1.638 .1400 2.80000000

 ABILITY | .02661537 .08946345 .297 .7737 .36600000

 MEDS | .10163069 .07017502 1.448 .1856 -.118424D-14

 FEDS | .00164437 .04464910 .037 .9715 .165793D-14

 SIBSS | .05916922 .06901801 .857 .4162 -.592119D-16

In the first set of results, the first coefficient vector is

**b**1 = (**X**1′**M**2**X**1)-1**X**1′**M**2**y** and **b**2 = (**X**2′**M**1**X**2)-1**X**2′M1**y**

In the second regression, the second set of regressors is M1X2, so

**b**1 = (**X**1′**M**12 **X**1)-1**X**1′**M**12**y** where **M**12 = **I** – (**M**1**X**2)[(M1**X**2)′(**M**1**X**2)]-1(**M**1**X**2)′

Thus, because the “M” matrix is different, the coefficient vector is different. The second set of coefficients in the second regression is

**b**2 = [(**M**1**X**2)′**M**1(**M**1**X**2)]-1 (**M**1**X**2)**M**1**y** = (**X**2′**M**1**X**2)-1**X**2′**M**1**y** because **M**1 is idempotent.